Royal Academy of Science Project

ESTALMAT

Selection test

June 5th, 2018

Name:……
Surname:……
Birthdate:……**Grade:**……
Telephone number:……
School:……**email:**……

Read before you start working

Time to complete the test: 90 minutes.

First read all the exercises and then start with the ones you find easier. It is not necessary to answer the questions in the same order that are presented. The order is up to you.

We are not interested only in the solutions you find but also, especially in the reasoning that took you to them.

At the end you have to give us all the papers you've used during the test.

We want to know all the good ideas you've had trying to solve these tasks. Try to describe these ideas as clear as possible (we only need some brief indications). Remember we are also interested in partial solutions of the questions.

We wish you success.

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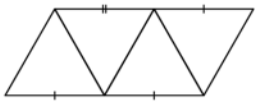
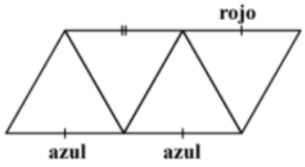
1. Colouring Triangles

Ana and Carlos play colouring the sides of the triangles of the drawing in blue, red or green. They must do it so that, in each of the four triangles, the three sides have different colour. First, Ana colours the three sides that have a single mark and then Carlos must say what colour can be the side that has a double mark, so that the previous condition is verified.



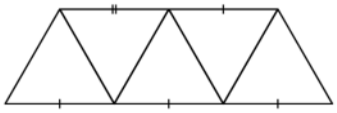
a. What colour Carlos can say, if Ana has used the colours indicated in the following figure? Reason your answer.



b. What colour could Carlos say if Ana initially painted those three sides in the same colour? Explain your answer.

c. Carlos says to Ana: "whatever colour you choose I will always be able to find a valid colour for the side with a double mark" Is it true what Carlos says? In any case, is there more than one colour that Carlos can say? Explain your answers.

d. Then Ana says to Carlos: "If I put one more triangle I will be able to colour four sides so that you can't find a solution" Find one of these possibilities in which Ana colours the four sides with a single mark and Carlos can't finish the game because he can't find a colour for the side with a double mark.



2. THE NUMBERS MACHINE

We have a machine in which we can only enter natural numbers 1; 2; 3; etc.

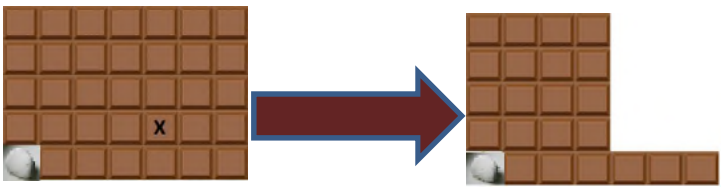
This machine first squares the number we have entered and then, with the result obtained, adds up all the digits as many times as necessary until we have a single digit and returns us this digit.

For example if we enter number 16, first squares it, $16^2 = 16x16 = 256$ and then adds up its digits 2+5+6=13. Since the outcome must have a single digit, it adds up the digits of the number 13 and gets 1+3=4. Therefore, if we enter 16 it returns us number 4.

a. What number will it return us if we enter number 26?
b. Can you give an example of a two digit number that you can enter in the machine so that the returned number will be 7?
c. We enter a number in the machine and it returns us number 9. If we now enter its consecutive number, can the machine return us number 9 too? If the answer is yes, explain why. If it's no, explain how many more numbers we have to advance so that the number returned by the machine can be a 9 again.
d. Can you enter any number in the machine so it returns number 5? Reason your answer.
e. What result will the machine return if we enter the number 201820182018? Reason your answer.
f. If we modify the machine and in the first step it cubes the number instead of squaring it (25 cubed is 25x25x25), can you tell what result it will return if we start with the number 20182018? Reason your answer.
g. Would you be able to build an original machine in such a way that when you enter any natural number 1, 2, 3, 4, etc., it will always return number 1 or 2 or 3?

3. CHOMP GAME

Chomp is a game for two players. In each turn, players eat pieces of a rectangular chocolate bar that is divided into squares. Before starting the game, the lower left square is replaced by a stone, as you can see in the next picture.
Each player, in their turn, chooses a small square of chocolate that hasn't been eaten yet and eats it along with all the squares located on the right and above the selected square. For example if a player chooses the square marked with an X in the picture, at the end of his turn the chocolate bar will look like the one on the right of the arrow.



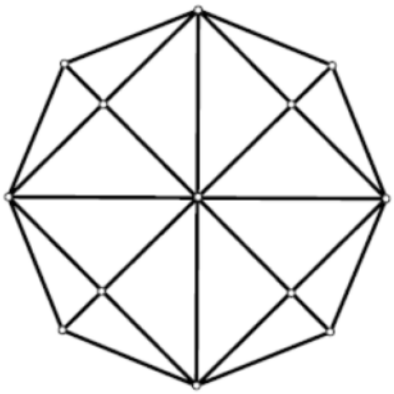
The player who eats the last chocolate square is the winner, and the looser gets the stone.
According to the rules and supposing that the two players always make the best play and never make mistakes, answer the following questions:

a. In a 2x2 chocolate bar (2 rows, 2 columns), what square has to choose the first player in its first turn to win the game no matter what the second player does?
b. In a 3x3 chocolate bar, the first player in its first turn chooses the lower right square. Who will be the winner supposing that both players play at their best.
c. Imagine that you are playing in a 3x3 chocolate bar and you're the first top player. Find a way to play that makes you win no matter what the other player does. Could you start in a different square and still keep winning?
d. Imagine a chocolate bar of any size you want. If you are the first player and it's your first turn, is there any square that always makes you loose if your opponent is a good player?
e. If the first person to play is a good player, do you think he will be able to win no matter the size of the chocolate bar? If you think the answer is yes, explain your reasoning. If you think the answer is no, give an example of a chocolate bar so that the second player always win.

4. COUNTING TRIANGLES ACORDING TO THEIR SIDES

Remember that according to their sides, triangles can be:

- Acute triangle, has three acute angles
- Right triangle, has one angle =90º
- Obtuse triangle, has one obtuse angle.



How many triangles of each type, with vertex in the points, do you see in this picture?

Acute triangles_____
Right triangles_____
Obtuse triangles _____

Explain the way you've count them.